

Theoretical and experimental investigation of onset characteristics of standing-wave thermoacoustic engines based on thermodynamic analysis



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ABSTRACT

A simplified physical model for calculating the onset temperature ratio and the frequency of a standing wave thermoacoustic engine (SWTE) in the time domain is built based on thermodynamic analysis. Coefficients of transient pressure drop and heat transfer are first deduced from linear thermoacoustic theory. By numerical computation, the evolutions of the pressure amplitude and the spectrum characteristics during the onset process are presented. Furthermore, the effects of stack spacing, charge pressure, and resonator length on the onset temperature ratio and the frequency are calculated. Relatively good agreement between the computational and the experimental results has been achieved, which validates the model for calculating the onset characteristics.

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1. Introduction

A thermoacoustic engine converts heat into acoustic power by thermoacoustic effects. It has been attracting more and more attention for its high efficiency, simple configuration, high reliability, and capability to utilize low temperature thermal energy [1,2]. Many efforts have been made to reveal the operating principle of thermoacoustic phenomena. Linear thermoacoustic theory, proposed by Rott [3], is now widely accepted. Only paying attention to the steady state and simplifying the analyzing process, Swift et al. converted the differential equations for thermoacoustics into algebraic equations by introducing complex notation [4,5]. The time terms are eliminated in the linearized equations, which means that the equations cannot apply to the simulation in time domain directly. However, the time evolution process is indispensable for understanding the mechanism of thermoacoustic onset process.

In recent years, nonlinear thermoacoustic theories have been developed rapidly. The quasi-one-dimensional nonlinear models proposed by Yuan and Karpov [6,7] and the two-dimensional model proposed by Hamilton [8] are capable of simulating the self-excited oscillation process of a SWTE in time domain. Meanwhile, computational fluid dynamics (CFD) has been used to simulate

thermoacoustic engines [9–12]. It is worth noting that the transient characteristics of pressure drop and heat transfer are necessary for the above numerical simulations in time domain to achieve accurate results. However, there are few methods that supply sufficient and reliable data for the oscillating flow in the stack or regenerator, due to the limited theoretical [13–16] and experimental [17–19] studies. Therefore, most nonlinear models have had to make use of the time-averaged data of pressure drop and heat transfer coefficients to calculate the energy and momentum exchange terms.

A thermodynamic analysis method was proposed by de Waele [20] to reveal the onset conditions of a traveling wave thermoacoustic engine, where all the components of the engine were converted analogously. To simplify the solution, an ideal regenerator was assumed, in which the energy equation was not required. The analytical expressions for the onset temperature and frequency were obtained. However, due to the intrinsically imperfect heat transfer in the stack of a SWTE, the method cannot be applied directly to SWTEs.

In this study, all components of a SWTE are converted analogously to build a full model [21]. The energy equation is introduced to describe the imperfect heat transfer characteristics in the stack. Particularly, six coefficients were deduced from the linear thermoacoustic theory to calculate the transient pressure drop and heat transfer in the stack. The whole process of building the model is presented in detail here. By solving the differential equation set

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Nomenclature

A	area, m^2	θ_Q	a coefficient for heat transfer, s/rad
c	sound velocity, m/s	θ_T	a coefficient for heat transfer, s/rad
c_p	specific heat at constant pressure, J/(kg K)	θ_v	a coefficient for pressure drop, s/rad
C	flow conductance, $\text{m}^3/(\text{Pa s})$	ρ	density, kg/m^3
D	diameter, m ; a coefficient for pressure drop, $\text{kg rad}/(\text{m}^4 \text{s})$	δ_κ	thermal penetration depth, m
f	frequency, Hz	δ_v	viscous penetration depth, m
f_κ	spatial average thermal function	τ	temperature ratio
f_v	spatial average viscous function	ω	angular frequency, rad/s
G	a coefficient for heat transfer, J/K		
H	a coefficient for heat transfer, $\text{J rad}/(\text{K s})$	Prefix	
L	length, m	δ	deviation from average value
$\dot{m}$	mass flow rate, kg/s	Δ	difference value
m	mass, kg	Subscript	
p	pressure, Pa	b	buffer behind the resonator
Pr	Prandtl number	c	cool end
Q	rate under which heat is transferred, W	g	gas
q	heat flow per unit area, W/m^2	h	hot end
R	gas constant, J/(kg K)	m	mean
T	temperature, K	O	Orifice
u	velocity, m/s	R	resonator
U	volume flow rate, m^3/s	s	stack
V	volume, m^3	0	time-average value for oscillation
y	perpendicular distance from plate in the stack, m	1	First order component for oscillation parameter
y_0	half of the stack spacing, m		
ϕ	porosity of the stack	Special symbols	
γ	specific heat ratio	$\langle \rangle$	spatial average perpendicular to axial direction
μ	dynamic viscosity, Pa s		

numerically, the onset processes of the SWTE are acquired. The onset temperature ratio and oscillation frequency of the SWTE are obtained and further verified by experiments.

2. Theoretical analysis of a SWTE

Fig. 1 shows the schematic of a SWTE consisting of a hot tube, a hot heat exchanger (HHX), a stack, a cold heat exchanger (CHX), a resonator, and a buffer. Except for the resonator, the lengths of other components in the SWTE are usually much smaller than the acoustic wavelength. Thus, all the components except for the resonator are treated by lumped parameter method, in which compliance and resistance components are converted into volumes with uniform pressures and an orifice together with a buffer, respectively. Similar to de Waele's treatment about the resonator [20], the resonator together with the buffer is converted into two volumes with uniform pressures and an in-between moving mass. Because of the intrinsic imperfection of the thermodynamic process occurring in the stack of the SWTE, coefficients deduced from linear thermoacoustic theory are used to calculate the transient pressure drop and the heat transfer in the stack.

To simplify the solution and reflect the real conditions, theoretical analysis of the SWTE in this study is based on the following assumptions:

- (1) The pressure amplitude in the SWTE is much smaller than the charge pressure.
- (2) The solid temperature profile in the stack is linear and independent of time.
- (3) For the hot tube and the buffer, only the compliance effect is considered. For the resonator, only the inertance effect is considered.
- (4) The void volumes of the heat exchangers HHX and CHX are ignored.
- (5) Except for the stack, the HHX, and the CHX, the other components are considered to be adiabatic.
- (6) There is no acoustic streaming occurring in the SWTE.
- (7) Working medium is assumed to be ideal gas.

It should be pointed out that since the inertance effect of a short resonator is not dominant any more, omitting the compliance effect and resistance of the resonator for a simplified solution may lead to a relatively large deviation when the resonator is short.

As the resonator is so long that it cannot be considered as a compartment with uniform pressure, the components of the SWTE should be converted analogously to simplify the analysis. The converted SWTE is shown in Fig. 2. With the above assumption (3), the hot tube and the buffer can be regarded as compartments with uniform pressure. The resonator is replaced by a solid piston with a

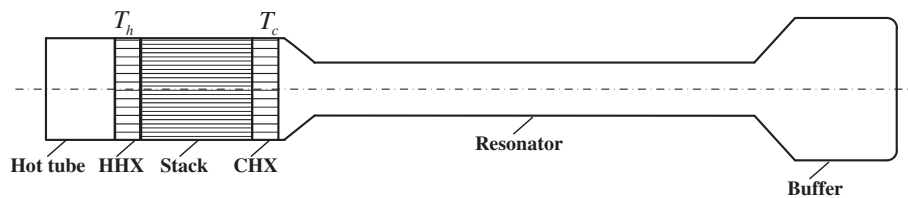


Fig. 1. Schematic of a SWTE.

mass m_R . The orifice together with a buffer is a load of the SWTE, which may represent the dissipation in the resonator and the buffer behind the resonator.

The lengths of the resonators at the two sides of the piston are L_R and L'_R , respectively. Let m_R equals the mass of gas contained in the resonator in front of the piston, i.e. $m_R = \rho_0 L_R A_R$.

- (a) If the resonator is connected to a buffer with infinite volume, the resonance angular frequency of the gas spring system combined with the piston and the compartments meets the following equation:

$$\omega^2 = \frac{\gamma p_0}{\rho_0 L_R^2}. \quad (1)$$

For a tube with an open end, the resonance angular frequency ω' is:

$$\omega' = \frac{\pi c}{2L_{ac}}, \quad (2)$$

where L_{ac} is the length of the resonator.

Making the resonance angular frequencies before and after the conversion the same, i.e. $\omega = \omega'$, then:

$$L_R = \frac{2}{\pi} L_{ac}. \quad (3)$$

- (b) If the outlet of the resonator is closed, the resonance angular frequency of the gas spring system should meet the following equation:

$$\omega^2 = \frac{\gamma p_0}{\rho_0 L_R} \left(\frac{1}{L_R} + \frac{1}{L'_R} \right). \quad (4)$$

For a tube with close ends, the resonance angular frequency ω' is:

$$\frac{\omega'}{2\pi} = \frac{c}{2L_{ac}}. \quad (5)$$

Making $\omega = \omega'$ and combining with Eq. (3), then:

$$L'_R = \frac{1}{3} L_R = \frac{2}{3\pi} L_{ac}. \quad (6)$$

The volume of the buffer after the conversion is:

$$V_b = V'_R + V'_b, \quad (7)$$

where V'_b is the volume of the buffer before the conversion.

Let $T_m = (T_h + T_c)/2$, the pressure in the hot tube $p_h = p_0 + \delta p_h$, the pressure in the ambient end $p_c = p_0 + \delta p_c$, the pressure and the temperature in the stack $p_s = p_0 + \delta p_s$ and $T_s = T_m + \delta T_s$ respectively. The pressure and the temperature in the stack are all space-averaged time-dependent parameters.

As the characteristic dimension of the hot tube is usually much larger than the thermal penetration depth of the working gas, the thermodynamic processes in the hot tube can be taken as adiabatic. The volume flow rate U_h is:

$$U_h = -\frac{1}{w_h} \frac{d\delta p_h}{dt}, \quad (8)$$

where $w_h = \frac{\gamma p_h}{V_h} \approx \frac{\gamma p_0}{V_h}$.

The parallel-plate type stack is used in this model for its high efficiency [5]. According to the continuity equation of the stack:

$$\dot{m}_h - \dot{m}_c = \frac{dm_s}{dt} \approx m_{s,0} \left(\frac{1}{p_0} \frac{dp_s}{dt} - \frac{1}{T_m} \frac{dT_s}{dt} \right). \quad (9)$$

Let $\tau = \frac{T_h}{T_c}$, then $\rho_c \approx \tau \rho_h$. Dividing Eq. (9) by ρ_h yields:

$$U_h - \tau U_c = \frac{m_{s,0}}{\rho_h} \left(\frac{1}{p_0} \frac{dp_s}{dt} - \frac{1}{T_m} \frac{dT_s}{dt} \right). \quad (10)$$

The pressure drop of a thermoacoustic component derived by Swift [5] can be expressed as:

$$\frac{dp_1}{dx} = -\frac{i\omega\rho_0}{1-f_v} u_1 = \frac{i\omega\rho_0/A}{1-f_v} U_1. \quad (11)$$

In time domain, the pressure drop along the stack can be expressed as:

$$\delta p_h - \delta p_s = D_h \left(1 + \theta_{vh} \frac{\partial}{\partial t} \right) U_h, \quad (12)$$

$$\text{and } \delta p_s - \delta p_c = D_c \left(1 + \theta_{vc} \frac{\partial}{\partial t} \right) U_c, \quad (13)$$

where the coefficients D and θ_v indicate the pressure drop dependence of volume flow rate U and first-order derivation of U , respectively.

Using the short stack approximation [5], it gives

$$D_h(1 + i\omega\theta_{vh}) = \frac{1}{2} \frac{i\omega\rho_{mh}}{(1-f_{vh})} \frac{L_s}{A_g}, \quad (14)$$

$$\text{and } D_c(1 + i\omega\theta_{vc}) = \frac{1}{2} \frac{i\omega\rho_{mc}}{(1-f_{vc})} \frac{L_s}{A_g}, \quad (15)$$

where $A_g = \phi A_s$ is the gas area of the stack cross section.

Separating the real and the imaginary parts, the coefficients D_h , D_c , θ_{vh} , and θ_{vc} can be calculated.

From the equations derived by Swift [5], neglecting the small terms involving the conductivity and the heat capacity of the solid which will not affect much on the results, the temperature perturbation in the channel can be written as:

$$T_1 = \frac{p_1}{\rho_0 c_p} \left[1 - \frac{\cosh((1+i)y/\delta_\kappa)}{\cosh((1+i)2y_0/\delta_\kappa)} \right] + \frac{1}{\omega^2 \rho_0} \frac{dp_1}{dx} \times \frac{dT_w}{dx} \left[\frac{\Pr}{\Pr-1} \frac{\cosh((1+i)y/\delta_v)}{\cosh((1+i)2y_0/\delta_v)} - \frac{1}{\Pr-1} \frac{\cosh((1+i)y/\delta_\kappa)}{\cosh((1+i)2y_0/\delta_\kappa)} - 1 \right] \quad (16)$$

Using Eq. (16), the mean temperature over the channel width is:

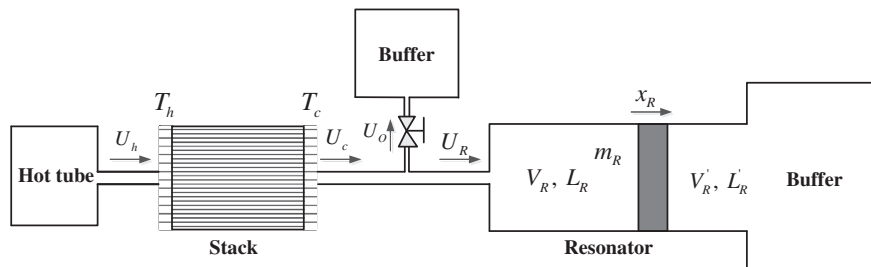


Fig. 2. Schematic of the SWTE after an analogy conversion.

$$\langle T_1 \rangle = (1 - f_\kappa) \frac{p_1}{\rho_0 c_p} + \frac{1}{\omega^2 \rho_0 (1 - \text{Pr})} \times [\text{Pr}(1 - f_v) - (1 - f_\kappa)] \times \frac{dT_w}{dx} \frac{dp_1}{dx}. \quad (17)$$

According to Eq. (16), the heat flux into the solid boundaries is:

$$-\vec{q} \cdot \vec{n} = \rho_0 c_p y_0 \left[i\omega f_\kappa \frac{p_1}{\rho_0 c_p} + \frac{f_v - f_\kappa}{(1 - f_v)(1 - \text{Pr})} \frac{dT_w}{dx} u_1 \right]. \quad (18)$$

Combining Eqs. (17) and (18), the heat transferred from the boundaries is:

$$Q = \rho c_p V_s \left[\frac{i\omega f_\kappa}{1 - f_\kappa} T_1 + \frac{1}{1 - \text{Pr}} \frac{dT_w}{dx} \left(\text{Pr} - 1 + \frac{1}{1 - f_v} - \frac{\text{Pr}}{1 - f_\kappa} \right) u_1 \right]. \quad (19)$$

From Eq. (19), the heat flow Q can be expressed as:

$$Q = Q(\Delta T) + Q(u), \quad (20)$$

where

$$Q(\Delta T) = H \left(1 + \theta_T \frac{\partial}{\partial t} \right) (T_s - T_w), \quad (21)$$

$$\text{and } Q(u) = \frac{dT_w}{dx} G \left(1 + \theta_Q \frac{\partial}{\partial t} \right) u. \quad (22)$$

Comparing with Eq. (19), we have:

$$H(1 + i\omega\theta_T) = \rho c_p V_s \frac{i\omega f_\kappa}{1 - f_\kappa}, \quad (23)$$

$$\text{and } G(1 + i\omega\theta_Q) = \frac{\rho c_p V_s}{1 - \text{Pr}} \left(\text{Pr} - 1 + \frac{1}{1 - f_v} - \frac{\text{Pr}}{1 - f_\kappa} \right). \quad (24)$$

According to the energy equation of the stack:

$$\begin{aligned} \frac{\partial}{\partial t} (\rho_s A_s L_s (c_v T_s + \frac{1}{2} u^2)) + \frac{\partial}{\partial x} [\rho_s A_s u L_s (c_p T_s + \frac{1}{2} u^2)] = \\ \frac{\partial}{\partial t} (c_v m_s T_s + \frac{1}{2} m_s u^2) + \frac{\partial}{\partial x} [(c_p \dot{m}_s T_s + \frac{1}{2} \dot{m}_s u^2) L_s] = -[Q(\Delta T) + Q(u)]. \end{aligned} \quad (25)$$

Neglecting the small terms of the kinetic energy term and $Q(u)$, Eq. (25) can be simplified as:

$$\frac{\partial (c_v m_s T_s)}{\partial t} = c_p \dot{m}_h T_h - c_p \dot{m}_c T_c - Q(\Delta T). \quad (26)$$

Let $m_s T_s = \frac{p_s V_s}{R}$, Eq. (26) can be expressed as:

$$\frac{c_v V_s}{R} \frac{dp_s}{dt} = c_p \dot{m}_h T_h - c_p \dot{m}_c T_c + Q(\Delta T). \quad (27)$$

Dividing by $\frac{c_v V_s}{R}$ yields:

$$\begin{aligned} \frac{dp_s}{dt} = \gamma R \dot{m}_h T_h - \gamma R \dot{m}_c T_c - \frac{\gamma R}{c_p V_s} H(1 + \theta_T \frac{\partial}{\partial t}) (T_s - T_w) \\ = \frac{\gamma p_0}{V_s} (U_h - U_c) - \frac{\gamma - 1}{V_s} H(1 + \theta_T \frac{\partial}{\partial t}) \delta T_s. \end{aligned} \quad (28)$$

Combining Eqs. (10) and (28) yields:

$$\frac{d\delta p_s}{dt} = a_1 U_h + a_2 U_c + a_3 \delta T_s, \quad (29)$$

$$\text{and } \frac{d\delta T_s}{dt} = b_1 U_h + b_2 U_c + b_3 \delta T_s, \quad (30)$$

where $a_1 = p_0 \chi_1 (\gamma p_0 + \chi_2 / T_h)$, $a_2 = -p_0 \chi_1 (\gamma p_0 + \chi_2 / T_c)$, $a_3 = -(\gamma - 1) p_0 H \chi_1$, $b_1 = p_0 T_m \chi_1 (\gamma - T_m / T_h)$, $b_2 = p_0 T_m \chi_1 (T_m / T_c - \gamma)$, $b_3 = -(\gamma - 1) H T_m \chi_1$, $\chi_1 = 1 / [p_0 V_s + (\gamma - 1) T_m H \theta_T]$, and $\chi_2 = (\gamma - 1) T_m^2 H \theta_T / V_s$.

The volume flow rate through the orifice U_o is:

$$U_o = C_o \delta p_c. \quad (31)$$

The orifice, together with the buffer, represents the dissipations in the resonator and the buffer, including viscous and thermal

dissipation. Thus, the thermal function in the resonator f_{κ_R} is used to calculate the flow conductance:

$$C_o = C' \text{Re}(f_{\kappa_R})^2. \quad (32)$$

The motion equation of the piston in the resonator yields:

$$m_R \frac{d^2 x_R}{dt^2} = A_R (\delta p_c - \delta p_b). \quad (33)$$

The volume of the resonator compartment in front of the piston can be expressed as: $V_R = V_{R0} + A_R x_R$. Eliminating x_R yields:

$$\frac{d^2 V_R}{dt^2} = a_R (\delta p_c - \delta p_b), \quad (34)$$

where $a_R = \frac{A_R^2}{m_R}$.

Taking the thermodynamic process in the resonator as adiabatic, it gives:

$$U_R = \frac{dV_R}{dt} + \frac{1}{w_R} \frac{d\delta p_c}{dt}, \quad (35)$$

where $w_R = \frac{\gamma p_c}{V_R} \approx \frac{\gamma p_0}{V_R}$.

According to the continuity equation:

$$U_c = U_R + U_o. \quad (36)$$

Combining Eqs. (35) and (36) to eliminate U_R yields:

$$\frac{d\delta p_c}{dt} = w_R \left(U_c - U_o - \frac{dV_R}{dt} \right). \quad (37)$$

The dynamic equations of the buffer reveal:

$$\frac{dV_b}{dt} = -\frac{1}{w_b} \frac{d\delta p_b}{dt}, \quad (38)$$

where $w_b = \frac{\gamma p_b}{V_b} \approx \frac{\gamma p_0}{V_b}$. For $dV_b = -dV_R$, Eq. (38) can be expressed as:

$$\frac{d\delta p_b}{dt} = w_b \frac{dV_R}{dt}. \quad (39)$$

For a SWTE with determined dimensions and operating parameters, there are eight unknown variables in the equation set formed by Eqs. 8, 12, 13, 29, 30, 34, 37, and 39. Thus, the equation set is closed and can be solved numerically.

3. Computational results

Combining the above governing differential equations, a detailed case of a self-designed SWTE using nitrogen of 1.0 MPa as the working gas has been provided. The main dimensions of the SWTE are shown in Table 1.

To solve the governing equation set, let $dV_R/dt = F_R$. Substituting it into Eqs. 34, 37, and 39 yields:

$$\frac{dF_R}{dt} = a_R (\delta p_c - \delta p_b), \quad (40)$$

$$\frac{d\delta p_b}{dt} = w_b F_R, \quad (41)$$

$$\text{and } \frac{d\delta p_c}{dt} = w_R (U_c - U_o - F_R). \quad (42)$$

Eqs. (8), (12), (13), (29), (30), (40)–(42) form a complete set of eight first-order differential equations, and classical Runge-Kutta integration is used as the algorithmic method. An initial perturbation is needed to excite the oscillation at the beginning. Values of $\delta p_c = 0$, $\delta p_h = 0$, $\delta p_s = 0$, $\delta p_b = 0$, $U_h = 0$, $U_c = 0$, $\delta T_s = 0$, and $F_R = 1.0 \times 10^{-5} \text{ m}^3/\text{s}$ are given as the initial values. The conductance coefficient C is set to be $1.0 \times 10^{-4} \text{ m}^3/(\text{Pa s})$. The cold end temperature of CHX is set to be

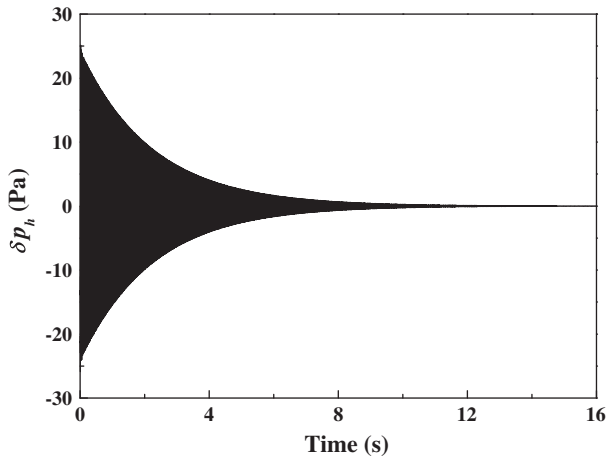
Table 1

Main dimensions of the self-designed SWTE.

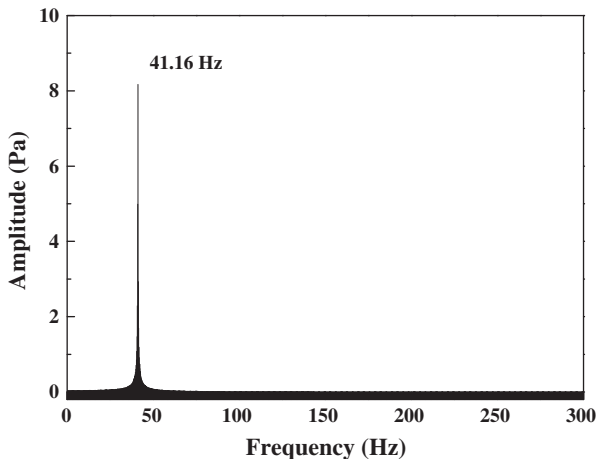
Components	Dimensions
Hot duct	$D_h = 59.5$ mm, $L_h = 60$ mm
Stack	$D_s = 59.5$ mm, $L_s = 110$ mm, $y_0 = 0.3$ mm, $\phi = 0.4$
Resonator	$D_R = 39$ mm, $L_{ac} = 2600$ mm
Buffer	$D_b = 85$ mm, $L_b = 500$ mm

300 K. An initial angular frequency ω is presumed to start the oscillation process and then the calculated oscillation angular frequency is used to get the final results.

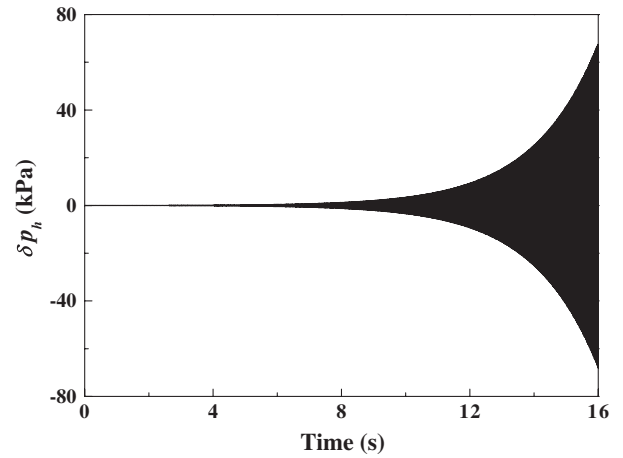
To calculate the onset temperature, the SWTE is simulated with different heating temperatures, increasing from 300 K step by step. When the pressure amplitude starts to grow, the heating temperature is taken as the onset temperature. Figs. 3 and 4 show the pressure evolution and the spectrum analysis in the hot tube with the temperature of the HHX of 450 K and 650 K, respectively. The given initial pressure amplitude damps gradually when T_h is 450 K, which indicates the heating temperature is lower than the required onset temperature. Whereas the pressure amplitude grows gradually when T_h is 650 K, which means the heating temperature is higher than the required onset temperature. According to the spectrum analysis diagram, the oscillation frequencies for the two heating temperatures are 41.16 Hz and 41.32 Hz,



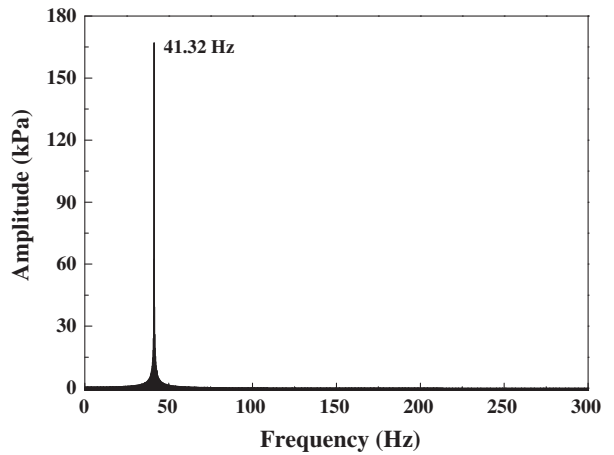
(a) Pressure evolution diagram in hot tube



(b) Spectrum diagram in hot tube



(a) Pressure evolution diagram in hot tube



(b) Spectrum diagram in hot tube

Fig. 4. Pressure evolution and spectrum diagrams in hot tube for $T_h = 650$ K and $C = 1.0 \times 10^{-4}$.

respectively. The oscillation frequency of the converted resonance piston and the buffer is $f = \sqrt{(w_R + w_b)} a_R / 2\pi = 42.82$ Hz, which is a little larger than the frequencies in simulations. This result indicates that the oscillation frequency of the SWTE system is mainly dependent on the resonant piston and buffer. The small difference between the oscillation frequencies of the SWTE system and the resonant piston and the buffer is due to the effects of the volumes of the hot tube and the stack. The operating frequency increases with the heating temperature because the physical properties of working gas in the hot tube and the stack change. As shown in the two spectrum diagrams, there is no obvious high order harmonic frequency, i.e., the energy is mostly focused in the fundamental acoustic component. In fact, there is usually a second-order harmonic frequency observed experimentally [22,23]. The difference is caused by replacing the resonator with a resonance piston in the model, which cannot show the harmonic characteristics.

Onset temperature ratio is a critical parameter to determine the heating temperature needed for a thermoacoustic engine to start. Typically, lower onset temperature ratio indicates that it is more likely for a thermoacoustic engine to utilize lower grade thermal energy. For a SWTE operates depending on a nonideal heat transfer between the solid and working gas, the stack spacing is one of the most important parameters in the design of a standing wave thermoacoustic engine. The effects of stack spacing on the onset temperature ratio of the SWTE are given in Fig. 5. As shown, the

Fig. 3. Pressure evolution and spectrum diagrams in hot tube for $T_h = 450$ K and $C = 1.0 \times 10^{-4}$.

onset temperature ratio varies with the stack spacing for a given charge pressure. There exists an optimal y_0 at which the lowest onset temperature ratio can be achieved corresponding to each charge pressure. The optimal y_0 decreases as the charge pressure increases under the present conditions. Furthermore, the onset temperature ratio varies with charge pressure for a given stack spacing. When the stack spacing is smaller than the optimal y_0 corresponding to the highest charge pressure, i.e. 2.1 MPa in the present case, a higher charge pressure results in a lower onset temperature ratio, otherwise the opposite. Table 2 shows the optimal values of y_0 with different charge pressures. It is shown that the lowest onset temperature ratios for different charge pressure are the same (about 1.708) under the present conditions. The ratio y_0/δ_κ for the lowest onset temperature ratio ranges from 1.28 to 1.33, which indicates that the optimal stack spacing is about 2.6–2.7 times the thermal penetration depth.

4. Experimental verification

In order to verify the above model, a SWTE was constructed and tested. The onset temperatures and pressure oscillations were measured at various charge pressures for different resonator lengths. The main dimensions of the SWTE are listed in Table 1, except for the resonator length. Seven resonators of different lengths, i.e. 0.6 m, 0.8 m, 1.1 m, 1.6 m, 2.1 m, 2.6 m, and 3.1 m, are tested in the experiments.

A calibrated piezoresistive pressure sensor with a total measuring error of about ± 643 Pa was installed to measure pressure. The heating temperatures were measured by a calibrated NiCr–NiSi thermocouple with a maximum measuring error of ± 3.5 K. Due to the difficulty in measuring the gas temperature directly, the solid temperature of the HHX is measured and used instead of the actual gas temperature. The solid temperature is higher than the gas temperature.

The effects of resonator length on the oscillation frequency with a charge pressure of 1.0 MPa are shown in Fig. 6. Both the calculated and experimentally obtained frequencies decrease as the resonator length increases. The calculated frequencies are larger than the experimental values because the compliance in the resonator is neglected. As stated before, the deviation between the calculated and experimental frequencies increases as the resonator length decreases because of the relatively more remarkable compliance effect for shorter resonators.

Fig. 7 shows the calculated and experimental results of the effects of charge pressure on the onset temperature ratio with

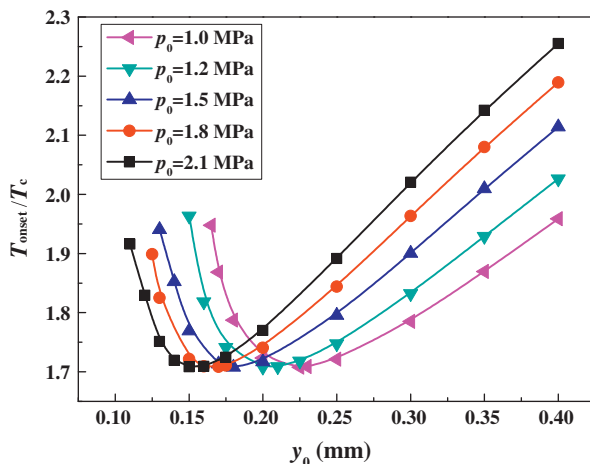


Fig. 5. Effects of stack spacing y_0 on onset temperature ratio with different charge pressures (calculated results).

Table 2

Optimal parameters to reach the lowest onset temperature ratio.

p_0 /MPa	1.0	1.2	1.5	1.8	2.1
Lowest onset temperature ratio	1.708	1.708	1.708	1.708	1.708
Optimal y_0 /mm	0.225	0.2	0.18	0.17	0.15
Optimal δ_κ /mm	0.172	0.155	0.139	0.127	0.117
Optimal y_0/δ_κ	1.31	1.29	1.29	1.33	1.28

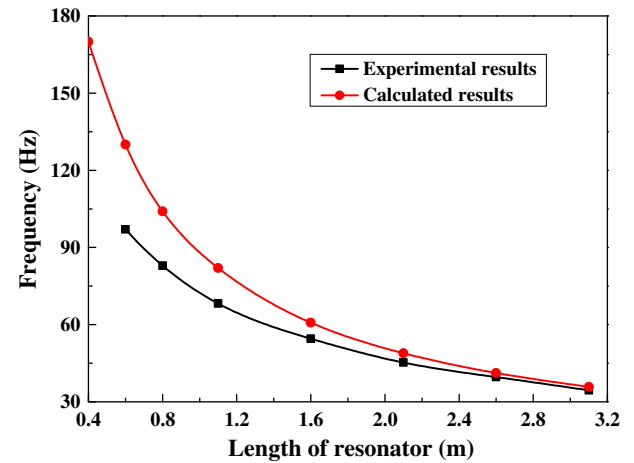


Fig. 6. Effects of resonator length on frequency with the charge pressure of 1.0 MPa.

different resonator length. The variation tendency of the calculated onset temperature ratio agrees well with what is obtained in experiments, especially when the length of the resonator is longer than 1.1 m. Both the calculated and the experimental results show that a lowest onset temperature can be achieved at a specific charge pressure for all the resonator lengths, which is defined as the optimal charge pressure here. The ranges of the optimal charge pressures of the simulations and experiments agree well. Similarly, there also exists an optimal resonator length to reach the lowest onset temperature for the SWTE, which is about 0.8 m in the calculations but around 1.1 m in the experiments.

The difference between the calculated and the experimental onset temperature ratios are shown in Fig. 8. The corresponding charge pressures of the maximum deviation are given above the points. As shown, both the maximum and the average deviations are within 15.5% when the resonator is longer than 1.6 m. For a resonator length of 3.1 m, the maximum deviation is only 7.4%, which

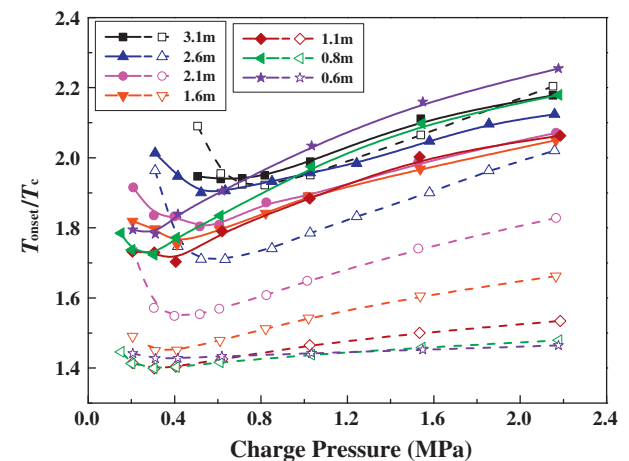


Fig. 7. Effects of charge pressure on onset temperature ratio with different resonator length when the stack spacing y_0 is 0.3 mm. The experimental and the computational results are denoted by the filled and open symbols, respectively.

shows an accurate prediction of the model for long resonator SWTEs. The deviations increase as resonator length decreases, which is similar to the frequency deviations. For the shortest resonator, the maximum deviation is near 35%. The deviations between calculated and measured onset temperature ratios are mainly due to two reasons. Firstly, the measured onset temperature is the solid wall temperature of the HHX, which is higher than actual gas temperature. Secondly, neglecting the compliance effect of the resonator results in the large deviation when the resonator is relatively short. Furthermore, it is found in Fig. 8 that the maximum deviations for resonators longer than 1.6 m occur at relatively low charge pressures and resonators shorter than 1.6 m at relatively high charge pressures. Although the inertance effect of long resonators is dominant compared to the compliance effect, the superiority of long resonators in inertance effect decreases when charge pressure gets smaller, which causes larger deviations because only inertance effect of the resonator are considered in the model. On the other hand, as the length of the resonator decreases, the compliance effect becomes more and more remarkable. For example, when the length of the resonator is 1.1 m and the charge pressure is 1.0 MPa, the magnitudes of the compliance impedance and the inertance impedance of the resonator are 4.24 MPa s/m^3 and 2.61 MPa s/m^3 , respectively. As the compliance impedance increases with the charge pressure, it is so remarkable for the short resonators at large charge pressures that neglecting it also causes larger deviations.

In order to further verify the model, experiments done by other researchers are also included for comparison. Arnott et al. built a SWTE, whose structure and dimensions were reported in detail [24]. The cold end heat exchanger and resonator were cooled by water to maintain the temperature at 293.15 K. The onset frequency and the onset temperature ratio of the SWTE are calculated and compared with the experimental results presented in Ref. [24], as shown in Figs. 9 and 10. Fig. 9 shows that the frequency of the SWTE is almost independent of charge pressure. The calculated frequencies are larger than the experimental values by about 12%. In Arnott's SWTE, the diameters of the hot tube and the resonator were almost the same, and the length of the resonator was about 1.29 m. Omitting the effects of compliance of the resonator and inertance of the hot tube causes the deviation between the calculated and experimental frequencies. Fig. 10 shows that the calculated onset temperature ratio agrees well with that of experiments in tendency. There exists an optimal charge pressure to achieve the lowest onset temperature. The maximum difference between the calculated and experimental onset temperature ratio is about 11.9%.

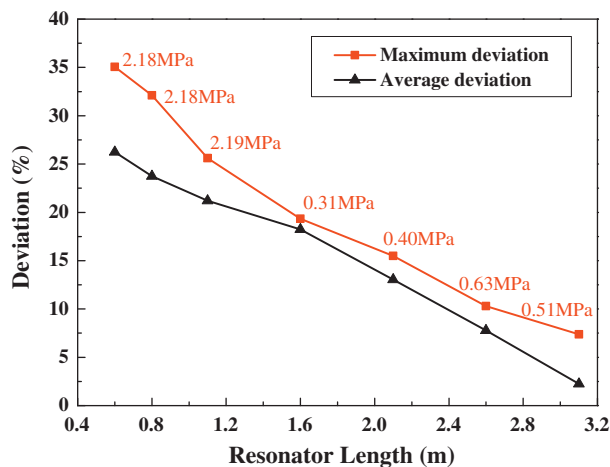


Fig. 8. Effects of resonator length on deviation of onset temperature ratio.

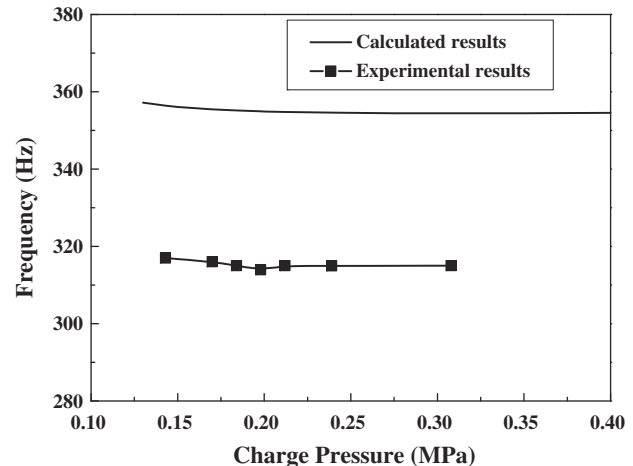


Fig. 9. Effects of charge pressure on onset frequency for Arnott's SWTE.

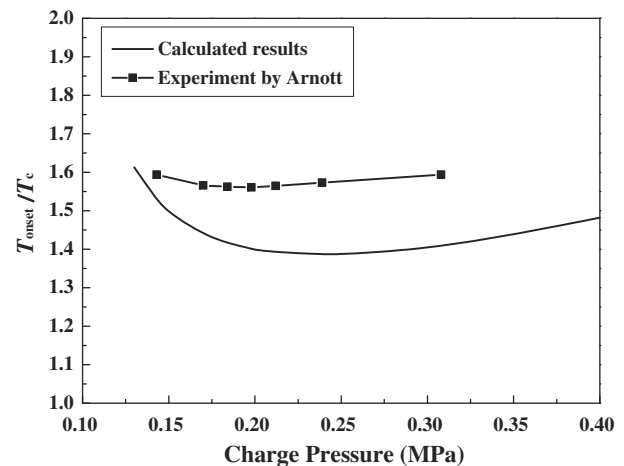


Fig. 10. Effects of charge pressure on onset temperature ratio for Arnott's SWTE.

5. Conclusion

This paper presents a simplified full physical model to simulate the onset characteristics of a SWTE based on thermodynamic analysis. Coefficients deduced from linear thermoacoustic theory are used to calculate the transient pressure drop and heat transfer in the stack. The growth conditions of pressure oscillation, evolutions of pressure amplitude, and the spectrum characteristics during the onset process are obtained. The calculated frequency and the onset temperature ratio agree well with the experimental results especially for long resonator SWTE.

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